Numerical Base Systems and Conversions between Them

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Part I

Background Information on Numerical Base Systems

Starting with Decimal We start with 10. That is the basis for what we normally call the everyday math used for money and counting. How many digits are there in a Base-10 numerical System? Well there are 10. These digits include 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. From these digits, we can make any number associated with our everyday math.

Why do I write of such trivial things? The answer is that not all people, modern or historical, nor all required mathematics from modern and historical times use base-10 numerical systems for everything. In this book I will provide you with some alternative numerical base systems that are both common and uncommon.

The more common base systems beside decimal are binary (associated with computers and logic), hexadecimal (also associated with computers) and vigesimal. The less common numerical base systems include ternary, quaternary, quinary, octal and alpha-numeric. Although I list some unusual base systems, there are many more that are not listed (i.e.; nonary or base-9, duodecimal or base-12, septemvigesimal or base-27, sexagesimal or base-60 used by the Sumerians).

Here is a chronological list of recorded decimal writers.

- c. 3500 - 2500 BC Elamites of Iran possibly used early forms of decimal system.
- c. 2900 BC Egyptian hieroglyphs show counting in powers of 10 (1 million + 400,000 goats, etc.) - see Ifrah, below
- c. 2600 BC Indus Valley Civilization, earliest known physical use of decimal fractions in ancient weight system: 1/20, 1/10, 1/5, 1/2. See Ancient Indus Valley weights and measures
- c. 1400 BC Chinese writers show familiarity with the concept: for example, 547 is written 'Five hundred plus four decades plus seven of days' in some manuscripts
- c. 1200 BC In ancient India, the Vedic text Yajur-Veda states the powers of 10, up to 10^{55}
- c. 400 BC Pingala - develops the binary number system for Sanskrit prosody, with a clear mapping to the base-10 decimal system
- c. 250 BC Archimedes writes the *Sand Reckoner*, which takes decimal calculation up to $10^{8\times10^{16}}$
- c. 100 - 200 The Satkhandagama written in India - earliest use of decimal logarithms
- c. 476 - 550 Aryabhata - uses an alphabetic cipher system for numbers that used zero
- c. 598 - 670 Brahmagupta - explains the Hindu-Arabic numerals (modern number system) which uses decimal integers, negative integers, and zero

- c. 920 - 980 Abu'l Hasan Ahmad ibn Ibrahim Al-Uqlidisi - earliest known direct mathematical treatment of decimal fractions.
- c. 1300 - 1500 The Kerala School in South India - decimal floating point numbers
- 1548/49 - 1620 Simon Stevin - author of *De Thiende* ('the tenth')
- 1561 - 1613 Bartholemaeus Pitiscus - (possibly) decimal point notation.
- 1550 - 1617 John Napier - use of decimal logarithms as a computational tool
- 1925 Louis Charles Karpinski - classic book *The History of Arithmetic* (Rand McNally and Company)
- 1959 Werner Buchholz *Fingers or Fists? (The Choice of Decimal or Binary representation)* (Communications of the ACM, Vol. 2 No. 12, pp3-11)
- 1974 Hermann Schmid *Decimal Computation* (ISBN 047176180X)
- 2000 Georges Ifrah *The Universal History of Numbers: From Prehistory to the Invention of the Computer* (ISBN 0-471-39340-1).

Binary System Binary Riddle: There are only 10 types of people on this planet. People that know binary and people that don't.

Where are the other 8 types? There aren't any more that 10_{binary} types. This becomes clear when we realize that binary has only 2 digits namely 0 and 1. This base-2 system is very useful for logic problems (true = 1, false =0) and for electronic circuits (on = 1, off = 0). Counting in binary is very easy: 0, 1, 10, 11, 100, 101, 110, 111, ...

The ancient Indian mathematician Pingala presented the first known description of a binary numeral system around 800 BC written in Hindu numerals. The numeration system was based on the Eye of Horus Old Kingdom numeration system.

A full set of 8 trigrams and 64 hexagrams, analogous to the 3-bit and 6-bit binary numerals, were known to the ancient Chinese in the classic text I Ching. Similar sets of binary combinations have also been used in traditional African divination systems such as If as well as in medieval Western geomancy.

An ordered binary arrangement of the hexagrams of the I Ching, representing the decimal sequence from 0 to 63, and a method for generating the same, was developed by the Chinese scholar and philosopher Shao Yong in the 11th century. However, there is no evidence that Shao understood binary computation.

In 1605 Francis Bacon discussed a system by which letters of the alphabet could be reduced to sequences of binary digits, which could then be encoded as scarcely visible variations in the font in any random text. Importantly for the general theory of binary encoding, he added that this method could be used with any objects at all: "provided those objects be capable of a twofold difference onely; as by Bells, by Trumpets, by Lights and Torches, by the report of Muskets, and any instruments of like nature."(See Bacon's cipher.)

The modern binary number system was fully documented by Gottfried Leibniz in the 17th century in his article Explication de l'Arithmtique Binaire. Leibniz's system used 0 and 1, like the modern binary numeral system.

In 1854, British mathematician George Boole published a landmark paper detailing a system of logic that would become known as Boolean algebra. His logical system proved instrumental in the development of the binary system, particularly in its implementation in electronic circuitry.

In 1937, Claude Shannon produced his master's thesis at MIT that implemented Boolean algebra and binary arithmetic using electronic relays and switches for the first time in history. Entitled A Symbolic Analysis of Relay and Switching Circuits, Shannon's thesis essentially founded practical digital circuit design.

In November of 1937, George Stibitz, then working at Bell Labs, completed a relay-based computer he dubbed the "Model K" (for "Kitchen", where he had assembled it), which calculated using binary addition. Bell Labs thus authorized a full research program in late 1938 with Stibitz at the helm. Their Complex Number Computer, completed January 8, 1940, was able to calculate complex numbers. In a demonstration to the American Mathematical Society conference at Dartmouth College on September 11, 1940, Stibitz was able to send the Complex Number Calculator remote commands over telephone lines by a teletype. It was the first computing machine ever used remotely over a phone line. Some participants of the conference who witnessed the demonstration were John Von Neumann, John Mauchly, and Norbert Wiener, who wrote about it in his memoirs.

Hexadecimal System This is not a system of counting developed by witches (German: die Hexen). It is a base-16 system which extends the usable digits of the normal decimal base system to 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F.

These numbers and symbols can also be written as 0_{16}, 1_{16}, 2_{16}, 3_{16}, 4_{16}, 5_{16}, 6_{16}, 7_{16}, 8_{16}, 9_{16}, A_{16}, B_{16}, C_{16}, D_{16}, E_{16}, and F_{16} for clarity and to separate it form Vigesimal.

Hexadecimal is primarily used in computing to represent a byte. Representing the 256 possible values has a number of problems: first, there are unprintable control characters; second, ASCII itself stops at 7 bits with the remainder being system-specific extensions; and finally, even if all characters in the machine's set were displayable as something, neither users nor input methods are generally prepared to handle 256 unique characters.

Some hexadecimal representations are indistinguishable from decimal representations (to humans and computers); therefore, some convention is usually used to flag them.

In typeset text, hexadecimal is often indicated by a subscripted suffix such as $5A3_{16}$, or $5A3_{HEX}$. In computer programming languages (which are nearly always plain text without such typographical distinctions as subscript and superscript),a wide variety of alternative notations are used to indicate hexadecimal numbers.

Vigesimal System Vigesimal is a base-20 system which extends the usable digits of decimal to include 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, G, H, I and J. These numbers and symbols can also be written as 0_{20}, 1_{20}, 2_{20}, 3_{20}, 4_{20}, 5_{20}, 6_{20}, 7_{20}, 8_{20}, 9_{20}, A_{20}, B_{20}, C_{20}, D_{20}, E_{20}, F_{20}, G_{20}, H_{20}, I_{20} and J_{20} for clarity and to separate it form Hexadecimal.

In many languages, especially in Europe, 20 is a base, at least with respect to the linguistic structure of the names of certain numbers (though a thoroughgoing consistent vigesimal system, based on the powers 20, 400, 8000 etc., is not generally used).

In East Asia, the Ainu language also uses a counting system that is based around the number 20. ”hotnep” is 20, ”wanpe etu hotnep” (ten more until two twenties) is 30, ”tu hotnep” (two twenties) is 40, ”ashikne hotnep” (five twenties) is 100. Subtraction is also heavily used, e.g. ”shinepesanpe” (one more until ten) is 9. Twenty was a base in the Maya and Aztec number systems. The Maya used the following names for the powers of twenty: kal (20), bak (20 = 400), pic (20 = 8,000), calab (204 = 160,000), kinchil (205 = 3,200,000) and alau (206 = 64,000,000). See also Maya numerals and Maya calendar, Mayan languages, Yucatec. The Aztec called them: cempoalli (1 20), centzontli (1 400), cenxiquipilli (1 8,000), cempoalxiquipilli (1 20 8,000 = 160,000), centzonxiquipilli (1 400 8,000 = 3,200,000) and cempoaltzonxiquipilli (1 20 400 8,000 = 64,000,000). Note that the ce(n/m) prefix at the beginning means ”one” (as in ”one hundred” and ”one thousand”) and is replaced with the corresponding number to get the names of other multiples of the power. For example, ome (2) poalli (20) = ompoalli (40), ome (2) tzontli (400) = ontzontli (800). Note also that the -li in poalli (and xiquipilli) and the -tli in tzontli are grammatical noun suffixes that are appended only at the end of the word; thus poalli, tzontli and xiquipilli compound together as poaltzonxiquipilli (instead of *poallitzontlixiquipilli). (See also Nahuatl language.)

Vigesimal was used in Europe. According to German linguist Theo Vennemann, the vigesimal system in Europe is of Basque (Vasconic) origin and spread from the so-called Vasconic languages to other European tongues, such as many Celtic languages, French and Danish.

According to Menninger, the vigesimal system originated with the Normans and spread through them to Western Europe, the evidence being that Celtic languages often use vigesimal counting systems. Others believe that this theory is unlikely, however.

- Twenty (vingt) is used as a base number in the French language names of numbers from 60 to 99. So for example, quatre-vingts, the French word for 80 literally means ”four twenties”, and soixante quinze, the word for 75 is literally ”sixty-fifteen.”).

- Twenty (tyve) is used as a base number in the Danish language names of numbers from 50 to 99. For example, Tres (short for tresindstyve) means 3 times 20, i.e. 60. For details, see Danish numerals.

- Twenty (ugent) is used as a base number in the Breton language names of numbers from 40 to 49 and from 60 to 99. For example, daou-ugent means 2 times 20, i.e. 40, and triwec'h ha pevar-ugent (literally ”three-six and four-twenty”) means 36 + 420, i.e. 98. However, 30 is tregont and not *dek ha ugent (”ten and twenty”), and 50 is hanter-kant (”half-hundred”).

- Twenty (ugain) is used as a base number in the Welsh language, although in the latter part of the twentieth century a decimal counting system has come to be preferred (particularly in the South), with the vigesimal system becoming 'traditional' and more popular in North Welsh. Deugain means 2 times 20 i.e. 40, trigain means 3 times 20 i.e. 60. Prior to the currency decimalisation in 1971, papur chwigain (6 times 20 paper) was the nickname for the 10 shilling (= 120 pence) note.

- Twenty (fiche) is used in an older counting system in Irish Gaelic, though most people nowadays use a decimal system, and this is what is taught in schools. Thirty is fiche is deich, literally twenty and ten. Forty is dh fhichid, literally two twenties (retained in the decimal system as daoichead). tr fichid is sixty (three twenties) and ceithre fichid is eighty (literally four twenties). Similarly, Scottish Gaelic has traditionally used a vigesimal system, with (fichead) being the word for twenty. A decimal system is now taught in schools.
- Twenty (njzet) is used as a base number in the Albanian language. The word for 40 (dyzet) means two times 20.
- Twenty (hogei) is used as a base number in the Basque language for numbers up to 100 (ehun). The words for 40 (berrogei), 60 (hirurogei) and 80 (laurogei) mean ”two-score”, ”three-score” and ”four-score”, respectively. The number 75 is called hirurogeita hamabost, lit. ”three-score-and ten-five”. The Basque nationalist Sabino Arana proposed three vigesimal digit systems to match the spoken language but they are mostly forgotten[citation needed].
- In the old British currency system, there were 20 shillings in a pound.
- In the imperial weight system there are twenty hundredweight in a ton.
- In English, counting by the score has been used historically, as in the famous opening of the Gettysburg Address ”Four score and seven years ago”, meaning eighty-seven (87) years ago. This method has fallen into disuse, however.
- Twenty (otsi) is used as a base number in the Georgian language. For example, 31 (otsdatertmeti) literally means, twenty-and-eleven. 67 (samotsdashvidi) is said as, three-twenty-and-seven.

Part II
Converting from one base to Decimal

1 Binary Conversions to Decimal

This is just a quick primer for those wanting to learn how to convert by hand between binary and decimal. Binary numbers are composed of only 0 and 1. Every positive decimal interger can be expressed in binary. By contrast, decinal numbers are composed of $0, 1, 2, ..., 9$.

Here is a binary number: 1101100

To convert easy binary numbers like this one to a decimal number we can define the placement of the numbers within a table.

Powers of 2	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	64	32	16	8	4	2	1
Binary	1	1	0	1	1	0	0
Results	64	32	0	8	4	0	0

1. Reading from left to right, we have a one in the Millions position which equals 64.
2. Next, there is a one in the Hundred Thousands position so we add 32 to 64 and get 96.
3. We have a zero in the Ten Thousands position so we do not add 16 and we are left with 96 still.
4. There is a one in the Thousands position so we add 8 to 96 and get 104.
5. The one in the Hundreds position adds 4 to 104 to get 108.
6. There are no ones in the Tens or the Ones position. Our final is 108.
7. Binary 0001101100 equals Decimal 108.

Now small numbers (less than 10 binary digits) are easy to convert using the table. Described below is a more universal way to convert binary numbers to decimal numbers.

Let us try to convert BIN 11011010010 to DEC.

We will first start a new table to make the conversion easier.

This table's rows can be extended for as many digits as is necessary to convert the binary number to decimal. The final tally number at the last digit is the decimal equivalent to the binary number (BIN 11011010010 is equivalent to DEC 1746).

Formula	Product	Tally
$0 \cdot 2^0$	0	0
$1 \cdot 2^1$	2	2
$0 \cdot 2^2$	0	2
$0 \cdot 2^3$	0	2
$1 \cdot 2^4$	16	18
$0 \cdot 2^5$	0	18
$1 \cdot 2^6$	64	82
$1 \cdot 2^7$	128	210
$0 \cdot 2^8$	0	210
$1 \cdot 2^9$	512	722
$1 \cdot 2^{10}$	1024	1746

2 Ternary Conversions to Decimal

Now we move into ternary base system. Ternary numbers are composed of only 0, 1 and 2. Every positive decimal interger can also be expressed using ternary notation.

Here is a ternary number: 22021_3

To convert this easy ternary number to a decimal number again define a placement table of the numbers such as is found in the table below.

Powers of 3	3^4	3^3	3^2	3^1	3^0
Decimal	81	27	9	3	1
Binary	2	2	0	2	1
Results	162	54	0	6	1

Now we just add up the results from the table above.

For Ternary 22021_3 the decimal equivalent is 223_{10}

Now small numbers (less than a few digits) are easy to convert using the table. When the numbers are large the following method is much easier.

Let us try to convert 22120021_3 to decimal.

We will first start a new table to make the conversion easier.

Formula	Product	Tally
$1 \cdot 3^0$	1	1
$2 \cdot 3^1$	6	7
$0 \cdot 3^2$	0	7
$0 \cdot 3^3$	0	7
$2 \cdot 3^4$	324	331
$1 \cdot 3^5$	243	574
$2 \cdot 3^6$	1458	2032
$2 \cdot 3^7$	4374	6406

This table's rows can be extended for as many digits as is necessary to convert the ternary number to decimal. The final tally number in the last Tally row is the decimal equivalent to the ternary number.

22120021_3 is equivalent to 6406_{10}

3 Quaternary Conversions to Decimal

Here we are at quaternary base-4 system. There is speculation as to how this numerical system developed (perhaps the knuckles of the hand). This system of counting involves the use of only 4 digits: 0, 1, 2 and 3. Let's start with an easy quaternary number.

Here is our quaternary number: 3102_{QUAT} or 3102_4

As we have done previously, we can use a quick conversion table to help us out.

Powers of 4	4^3	4^2	4^1	4^0
Decimal	64	16	4	1
Quaternary	3	1	0	2
Results	192	16	0	2

Again we add up the results from the table above. For Quaternary 3102_4 the decimal equivalent is 210_{10}. Now we can try to convert 32132103_4 to decimal using the following table.

Formula	Product	Tally
$3 \cdot 4^0$	3	3
$0 \cdot 4^1$	0	3
$1 \cdot 4^2$	16	19
$2 \cdot 4^3$	128	147
$3 \cdot 4^4$	768	915
$1 \cdot 4^5$	1024	1939
$2 \cdot 4^6$	8192	10131
$3 \cdot 4^7$	49152	59283

4 Quinary Conversions to Decimal

The quinary base-5 system was most likely developed based on the digits of the hand. This system of counting involves the use of only 5 digits: 0, 1, 2, 3 and 4.

Here is our quinary number: 4321_{QUIN} or 4321_5

Here is our quick conversion table to help us out.

Powers of 5	5^3	5^2	5^1	5^0
Decimal	125	25	5	1
Quinary	4	3	2	1
Results	500	75	10	1

Again we add up the results from the table above. For quinary 4321_5 the decimal equivalent is 586_{10}. Now we can try to convert 102344321_5 to decimal using the following table.

Formula	Product	Tally
$1 \cdot 5^0$	1	1
$2 \cdot 5^1$	10	11
$3 \cdot 5^2$	75	86
$4 \cdot 5^3$	500	586
$4 \cdot 5^4$	2500	3086
$3 \cdot 5^5$	9375	12461
$2 \cdot 5^6$	31250	43711
$0 \cdot 5^7$	0	43711
$1 \cdot 5^8$	390625	434336

5 Octal Conversions to Decimal

The octal base-8 system of counting involves the use of only 8 digits: 0, 1, 2, 3, 4, 5, 6 and 7.

Here is our octal number: 74561_{OCT} or 74561_8

Now we can convert 74561_8 to decimal using the following table.

Formula	Product	Tally
$1 \cdot 8^0$	1	1
$6 \cdot 8^1$	48	49
$5 \cdot 8^2$	320	369
$4 \cdot 8^3$	2048	2417
$7 \cdot 8^4$	28672	31089

6 Hexadecimal Conversions to Decimal

The hexadecimal base-16 system of counting involves the use of 16 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F.

Before we can convert from hexadecimal to decimal we must know that the numbers represented by letters correspond to decimal numbers as illustrated in the table below.

DEC	10	11	12	13	14	15
HEX	A	B	C	D	E	F

Here is our hexadecimal number: $FA23C6_{16}$

Now we can convert $FA23C6_{16}$ to decimal using the following table.

Formula	Product	Tally
$6(6) \cdot 16^0$	6	6
$C(12) \cdot 16^1$	192	198
$3(3) \cdot 16^2$	768	966
$2(2) \cdot 16^3$	8192	9158
$A(10) \cdot 16^4$	655360	664518
$F(15) \cdot 16^5$	15728640	16393158

7 Vigesimal Conversions to Decimal

The vigesimal base-20 system of counting involves the use of 20 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, G, H, I and J.

Before we can convert from vigesimal to decimal we must know that the numbers represented by letters correspond to decimal numbers as illustrated in the table below.

DEC	10	11	12	13	14	15	16	17	18	19
HEX	A	B	C	D	E	F	G	H	I	J

Here is our vigesimal number: $JI09_{20}$

Now we can convert $JI09_{20}$ to decimal using the following table.

Formula	Product	Tally
$9(9) \cdot 20^0$	9	9
$0(0) \cdot 20^1$	0	9
$I(18) \cdot 20^2$	7200	7209
$J(19) \cdot 20^3$	152000	159209

8 Alpha-Numerical Conversions to Decimal

The alpha-numeric base-36 system of counting involves the use of 36 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y and Z.

Before we can convert from alpha-numeric to decimal we must know that the numbers represented by letters correspond to decimal numbers as illustrated in the tables below.

DEC	10	11	12	13	14	15	16	17	18	19	20	21	22
HEX	A	B	C	D	E	F	G	H	I	J	K	L	M

DEC	23	24	25	26	27	28	29	30	31	32	33	34	35
HEX	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Here is our alpha-numeric number: $1PIZZA_{36}$

Now we can convert $1PIZZA_{36}$ to decimal using the following table.

Formula	Product	Tally
$A(10) \cdot 36^0$	10	10
$Z(35) \cdot 36^1$	1260	1270
$Z(35) \cdot 36^2$	45360	46630
$I(18) \cdot 36^3$	839808	886438
$P(25) \cdot 36^4$	41990400	42876838
$1(1) \cdot 36^5$	60466176	103343014

9 Distilled Equation for Converting any Base to Decimal

In the end we can distill all of the chapters where we have converted a base to decimal to the following equation:

$$\left(S_n \cdot B^{n-1}\right) + \left(S_{n-1} \cdot B^{n-2}\right) + \cdots + \left(S_1 \cdot B^0\right) \tag{1}$$

This equation is used by substituting the Base Number (i.e.; 2, 3, 4, 5, 8, 16, ...) for B and entering the highest digit for S_n and so on.

If we have a hexadecimal number of $F5A_{16}$ we substitute n with 3 for the number of digits, B with 16 and S_3 with F which equals 15 in decimal, $S_2 = 5$ and $S_1 = A$ (or $S_1 = 10$). So the final equation would be:

$$\left(15 \cdot 16^2\right) + \left(5 \cdot 16^1\right) + \left(10 \cdot 16^0\right) = 3930 \tag{2}$$

Equation (1) can be extended to include decimal notation (i.e.; $F1A.AE_{16}$) by modifying equation (1) to be:

$$\left(S_n \cdot B^{n-1}\right) + \left(S_{n-1} \cdot B^{n-2}\right) + \cdots + \left(S_1 \cdot B^0\right) + \left(S_{-1} \cdot B^{-1}\right) + \left(S_2 \cdot B^{-2}\right) + \cdots + \left(S_{-m} \cdot B^{-m}\right) \tag{3}$$

So to test this, let's use $IC2.A9_{20}$ and convert to decimal. We let $n = 3$ for the digits on the left of the decimal and $m = 2$ for the digits on the right of the decimal. Next we set $B = 20$ which is the base number system from which we want to convert to decimal.

$$\left(I(18) \cdot 20^2\right) + \left(C(12) \cdot 20^1\right) + \left(2(2) \cdot 20^0\right) + \left(A(10) \cdot 20^{-1}\right) + \left(9(9) \cdot 20^{-2}\right) = 7442.5225 \tag{4}$$

Part III
Converting from Decimal to Another Base

10 Decimal Conversions to Binary

Again we are back to the binary and decimal conversions. This time we will take a deimal number and convert it to a binary number.

Here is a decimal number: 234

To convert this to binary, we need to find the highest of the of these numbers 512, 256, 128, 64, 32, 16, 8, 4, 2, or 1 that is less than the decimal number we will convert. Again, with easy numbers (≤ 1023) we can quickly set up a table to help solve the conversion.

Power of 2	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Decimal	512	256	128	64	32	16	8	4	2	1
Binary	0	0	1	1	1	0	1	0	1	0
Result	0	0	128	64	32	0	8	0	2	0

1. First we find that 512 and 256 are bigger than 234 but 128 is not. So we place a one in the 128 column.
2. We subtract 128 from 234 and get 106. We put a one in the 64 column because it is smaller than 106.
3. We subract 64 from 106 and get 42. We put a one in the 32 column because it is smaller than 42.
4. We subract 32 from 42 and get 10. We put a zero in the 16 column because it is bigger than 10. We can put a 1 in the 8 column because it is smaller than 10.
5. We subract 8 from 10 and get 2. We put a zero in the 4 column because it is bigger than 2. We put a one the 2 column because it is equal to 2.
6. We subtract 2 from 2 and get zero. We put a zero in the 1 column because it is smaller than 0.
7. Decimal 234 equals Binary 11101010.

There is a more universal way to convert DEC to BIN that can be applied to larger numbers. This method is more involved but it can be used to convert any DEC number to BIN.

Let us use DEC 300. Write down each remainder for each division (explained below).

1. Divide DEC 300 by 2 (the base number of digits in BIN) and wriite down the remainder ($300/2 = 150$ remainder 0).
2. Now divide 150 by 2 ($150/2 = 75$ remainder 0).
3. We then divide 75 by 2 ($75/2 = 37$ remainder 1).
4. We then divide 37 by 2 ($37/2 = 18$ remainder 1).
5. We then divide 18 by 2 ($18/2 = 9$ remainder 0).
6. We then divide 9 by 2 ($9/2 = 4$ remainder 1).
7. We then divide 4 by 2 ($4/2 = 2$ remainder 0).
8. We then divide 2 by 2 ($2/2 = 1$ remainder 0).
9. Lastly, we divide 1 by 2 ($1/2 = 0$ remainder 1).

Finally, we put the remainders of each division together to form the binary equivalent of the decimal number. The remainder from our first division goes into the one place, the remainder from our second division goes into the tens place, third remainder goes into the hundreds place, and so on.

So our answer is DEC 300 = BIN 100101100

With this binary number (100101100), you can convert back to decimal using the table in the previous part of this document.

The universal convertion between DEC to BIN can be tabularized to make its application easier. Below is one way to tabularize the conversion. We will start with DEC 171.

Division	Quotient	Remainder
$171 \div 2$	85	1
$85 \div 2$	42	1
$42 \div 2$	21	0
$21 \div 2$	10	1
$10 \div 2$	5	0
$5 \div 2$	2	1
$2 \div 2$	1	0
$1 \div 2$	0	1

Thus, after filling in the table, we are left with placing the remainders into their proper positions. We work from the bottom of the table up to the top and place the number from left to right. The answer to the question:

The binary representation of decimal 171_{10} is 10101011_2

11 Decimal Conversions to Ternary

The use of ternary is very minimal. I include it here only for practice.

Here is a decimal number: 367_{10}

To convert this to terary, we need to find the highest of the of the following numbers that is less than the decimal number we will convert: 2187, 729, 243, 81, 27, 9, 3, or 1. Again, with easy decimal numbers ($\leq$ 2187) we can quickly set up a table to help solve the conversion (Table 11 below).

Power of 3	3^7	3^6	3^5	3^4	3^3	3^2	3^1	3^0
Decimal	2187	729	243	81	27	9	3	1
Ternary	0	0	1	1	1	1	2	1
Result	0	0	234	81	27	9	6	1

1. First we find the largest Power of 3 in decimal representation that is smaller than 367
2. We find that 2187 and 729 are larger than 367 but 243 is not. So we place a one (1) in the 367 column.
3. We subtract 243 from 367 and get 124.
4. We find the now find the largest Power of 3 in decimal representation that is smaller than 124 which is 81. So we place a one (1) in the 81 column.
5. We subtract 81 from 124 and get 43.
6. We put a one (1) in the 27 column because it is smaller than 43.
7. We subtract 27 from 43 and get 16.
8. We put a one (1) in the 9 column because it is smaller than 16.
9. We subtract 9 from 16 and get 7.
10. We put a one (1) in the 3 column because it is smaller than 7.
11. We subtract 3 from 7 and get 4.
12. Now here is a problem, 4 is still bigger than 3 so we remove the 1 from the 3 column and replace it with a 2.
13. Next we subtract 6 (which is $2 \cdot 3$) from 7 and get 1.
14. Lastly, put a one (1) in the 1 column because it is smaller than or equal to 1.
15. We subtract 1 from 1 and get 0.
16. Put the numbers together for the answer.
17. Decimal 367 equals 111121_{TER}.

There is a more universal way to convert decimal to ternary that can be applied to larger numbers. This method is more involved but it can be used to convert any decimal number to ternary.

Let us use DEC 1000. Write down each remainder for each division (explained below).

1. Divide DEC 1000 by 3 (the base number of digits in TER) and write down the remainder ($1000/3 = 333$ remainder 1).
2. Now divide 333 by 3 ($333/3 = 111$ remainder 0).
3. We then divide 111 by 3 ($111/3 = 37$ remainder 0).
4. We then divide 37 by 3 ($37/3 = 12$ remainder 1).
5. We then divide 12 by 3 ($12/3 = 4$ remainder 0).
6. We then divide 4 by 3 ($4/3 = 1$ remainder 1).
7. We then divide 1 by 3 ($1/3 = 0$ remainder 1).

Finally, we put the remainders of each division together to form the ternary equivalent of the decimal number. The remainder from our first division goes into the one place, the remainder from our second division goes into the tens place, third remainder goes into the hundreds place, and so on.

So our answer is DEC 1000 = TER 1101001

With this ternary number (1101001_3), you can convert back to decimal using the table in the previous part of this document.

The universal convertion between DEC to TER can be tabularized to make its application easier. Below is one way to tabularize the conversion. We will start with DEC 5821.

Division	Quotient	Remainder
$5821 \div 3$	1940	1
$1940 \div 3$	646	2
$646 \div 3$	215	1
$215 \div 3$	71	2
$71 \div 3$	23	2
$23 \div 3$	7	2
$7 \div 3$	2	1
$2 \div 3$	0	2

Thus, after filling in the table, we are left with placing the remainders into their proper positions. We work from the bottom of the table up to the top and place the number from left to right. The answer to the question:

What is the ternary representation of decimal 5821_10? 21222121_3

12 Decimal Conversions to Quaternary

We are going to avoid the hard method and go straight to the easy method to convert decimal to quaternary. This method is more involved but it can be used to convert any decimal number to quaternary.

Let us use the decimal number 10182_{10} and the table below.

Division	Quotient	Remainder
$10182 \div 4$	2545	2
$2545 \div 4$	636	1
$636 \div 4$	159	0
$159 \div 4$	39	3
$39 \div 4$	9	3
$9 \div 4$	2	1
$2 \div 4$	0	2

We work from the bottom of the table up to the top and place the number from left to right. The quaternary representation of 10182_{10} is 2133012_4

13 Decimal Conversions to Quinary

We are going to avoid the hard method and go straight to the easy method to convert decimal to quinary. This method is more involved but it can be used to convert any decimal number to quinary.

Let us use the decimal number 48561_{10} and the table below.

Division	Quotient	Remainder
$48561 \div 5$	9712	1
$9712 \div 5$	1942	2
$1942 \div 5$	388	2
$388 \div 5$	77	3
$77 \div 5$	15	2
$15 \div 5$	3	0
$3 \div 5$	0	3

We work from the bottom of the table up to the top and place the number from left to right. The quaternary representation of 48561_{10} is 3023221_5

14 Decimal Conversions to Octal

Let us use the decimal number 98765_{10} and the table below.

Division	Quotient	Remainder
$98765 \div 8$	12345	5
$12345 \div 8$	1543	1
$1543 \div 8$	192	7
$192 \div 8$	24	0
$24 \div 8$	3	0
$3 \div 8$	8	3

We work from the bottom of the table up to the top and place the number from left to right. The octal representation of 98765_{10} is 300715_8

15 Decimal Conversions to Hexadecimal

Before we can convert from decmial to hexadecimal we must know that the numbers represented by letters correspond to decimal numbers as illustrated in the table below.

DEC	10	11	12	13	14	15
HEX	A	B	C	D	E	F

Let us use the decimal number 998732_{10} and the table below.

Division	Quotient	Remainder
$998732 \div 16$	62420	$12(C)$
$62420 \div 16$	3901	$4(4)$
$3901 \div 16$	243	$13(D)$
$243 \div 16$	15	$3(3)$
$15 \div 16$	0	$15(F)$

We work from the bottom of the table up to the top and place the number from left to right. The hexadecimal representation of 998732_{10} is $F3D4C_{16}$

16 Decimal Conversions to Vigesimal

Before we can convert from decmial to vigesimal we must know that the numbers represented by letters correspond to decimal numbers as illustrated in the table below.

DEC	10	11	12	13	14	15	16	17	18	19
HEX	A	B	C	D	E	F	G	H	I	J

Let us use the decimal number 998732_{10} again.

Division	Quotient	Remainder
$998732 \div 20$	49936	$12(C)$
$49936 \div 20$	2496	$16(G)$
$2496 \div 20$	124	$16(G)$
$124 \div 20$	6	$4(4)$
$6 \div 20$	0	$6(6)$

We work from the bottom of the table up to the top and place the number from left to right. The vigesimal representation of 998732_{10} is $64GGC_{20}$

17 Decimal Conversions to Alpha-Numerical

Before we can convert from decmial to alpha-numeric we must know that the numbers represented by letters correspond to decimal numbers as illustrated in the tables below.

DEC	10	11	12	13	14	15	16	17	18	19	20	21	22
HEX	A	B	C	D	E	F	G	H	I	J	K	L	M

DEC	23	24	25	26	27	28	29	30	31	32	33	34	35
HEX	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Let us use the decimal number 9987329561_{10} for this number system.

Division	Quotient	Remainder
$9987329561 \div 36$	277425821	5(5)
$277425821 \div 36$	7706272	$29(T)$
$7706272 \div 36$	214063	4(4)
$214063 \div 36$	588	3(3)
$588 \div 36$	16	$12(C)$
$16 \div 36$	0	$16(G)$

We work from the bottom of the table up to the top and place the number from left to right. The alpha-numeric representation of 9987329561_{10} is $GC34T5_{36}$

Part IV
Tables

DEC	0	1	2	3	4	5	6	7	8	9
BIN	0	1	10	11	100	101	110	111	1000	1001
TER	0	1	2	10	11	12	20	21	22	100
QUAT	0	1	2	3	10	11	12	13	20	21
QUIN	0	1	2	3	4	10	11	12	13	14
OCT	0	1	2	3	4	5	6	7	10	11
HEX	0	1	2	3	4	5	6	7	8	9
VIG	0	1	2	3	4	5	6	7	8	9
ALPHA	0	1	2	3	4	5	6	7	8	9

DEC	10	11	12	13	14	15	16	17	18	19
TER	101	102	110	111	112	120	121	122	200	201
QUAT	22	23	30	31	32	33	100	101	102	103
QUIN	20	21	22	23	24	30	31	32	33	34
OCT	12	13	14	15	16	17	20	21	22	23
HEX	A	B	C	D	E	F	10	11	12	13
VIG	A	B	C	D	E	F	G	H	I	J
ALPHA	A	B	C	D	E	F	G	H	I	J

DEC	20	21	22	23	24	25	26	27	28	29
HEX	14	15	16	17	18	19	1A	1B	1C	1D
VIG	10	11	12	13	14	15	16	17	18	19
ALPHA	K	L	M	N	O	P	Q	R	S	T

DEC	30	31	32	33	34	35	36	37	38	39
HEX	1E	1F	20	21	22	23	24	25	26	27
VIG	1A	1B	1C	1D	1E	1F	1G	1H	1I	1J
ALPHA	U	V	W	X	Y	Z	10	11	12	13

Part V
Thanks

First I would like to thank the creator of TEX Donald Knuth. From which LATEXwas created. LATEXwas originally written in the early 1980s by Leslie Lamport at SRI International. It has become the dominant method for using TEX. The current version is LATEX2ϵ.

Wikipedia (http://wikipedia.org/) has been integral in finding the history of the numerical base systems.