square root

(algorithm)

Definition: This describes a "long hand" or manual method of calculating or extracting square roots. Calculation of a square root by hand is a little like long-hand division.

Suppose you need to find the square root of 66564. Set up a "division" with the number under the radical. Mark off pairs of digits, starting from the decimal point. (Here the decimal point is a period (.) and a comma (,) marks pairs of digits.)

```
    ___________
\/   6,65,64.
```

Look at the leftmost digit(s) (6 in this case). What is the largest number whose square is less than or equal to it? It is 2, whose square is 4. Write 2 above, write the square below and subtract.

```
    __2________
\/   6,65,64.
    -4
   ----
     2
```

Now bring down the next two digits (65). The next "divisor" is double the number on top (2x2=4) and some other digit in the units position (4_).

```
    __2________
\/   6,65,64.
    -4
    -----
4_ )  265
```

What is the largest number that we can put in the units and multiply times the divisor and still be less than or equal to what we have? (Algebraically, what is d such that d × 4d ≤ 265?) It looks like 6 might work (since 6 * 40 = 240), but 6 is too big, since 6 * 46 = 276.

```
    __2__6_____
\/   6,65,64.
    -4
    -----
46 )  265
      276    TOO BIG
```

So try 5 instead.

```
    __2__5_____
\/   6,65,64.
    -4
    -----
```

```
45 ) 265
    -225
    -------
      40
```

Repeat: bring down the next two digits, and double the number on top (2x25=50) to make a "divisor", with another unit.

```
  __2__5_____
\/  6,65,64.
   -4
   -----
45 ) 265
    -225
    -------
50_ ) 4064
```

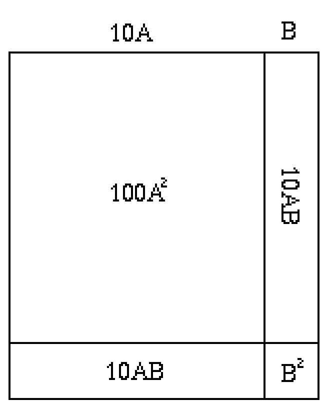
It looks like 8 would work. Let's see.

```
  __2__5__8__
\/  6,65,64.
   -4
   -----
45 ) 265
    -225
    -------
508 ) 4064
     -4064
     ------
         0
```

So the square root of 66564 is 258. You can continue for as many decimal places as you need: just bring down more pairs of zeros.

Why does this work?

Consider $(10A + B)^2 = 100A^2 + 2 \times 10AB + B^2$ and think about finding the area of a square. Remember that $10A + B$ is just the numeral with B in the units place and A in the higher position. For 42, A=4 and B=2, so $10 \times 4 + 2 = 42$.



The area of the two skinny rectangles is $2 \times 10A \times B$. The tiny square is B^2. If we know A and the area of the square, S, what B should we choose?

We previously subtracted A^2 from S. To scale to $100A^2$, we bring down two more digits (a factor of 100) of the size of S. We write down twice A (2A), but shifted one place to leave room for B ($10 \times 2A$ or $2 \times 10A$). Now we add B to get $2 \times 10A + B$. Multiplying by B gives us $2 \times 10AB + B^2$. When we subtract that from the remainder (remember we already subtracted A^2), we have subtracted exactly $(10A + B)^2$. That is, we have improved our knowledge of the square root by one digit, B.

We take whatever remains, scale again by 100, by bringing down two more digits, and repeat the process.

See also *cube root*.

Note: In computers and hand-held calculators, square root, sine, cosine, and other transcendental functions are calculated with sophisticated functions, such as Taylor series, CORDIC, or Newton-Raphson method, sometimes called Newton's method. This lesson explains, for instance, possible difficulties in convergence.

Author: PEB

Implementation

GAMS Class C2 has many implementations of powers, roots, and reciprocals (C and Fortran). Many variations (C and Assembler) for caching, pipelined processing, etc.

More information

Another geometric justification.

Go to the Dictionary of Algorithms and Data Structures home page.

If you have suggestions, corrections, or comments, please get in touch with Paul E. Black.

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